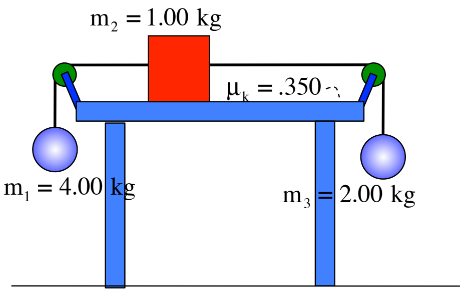
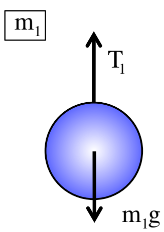
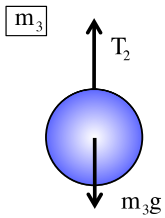
Problem 5.46

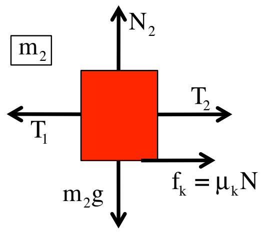
a.) F.b.d. for each mass?



Note that the tensions are different on either side of m_2, but are the same on either side of each pulley. With that, we can write:







b.) Determine the acceleration of the system:

Quick and Dirty: Assume the forces that make m_2 accelerate to the left are positive (which means that the acceleration of the system in this case will be positive and to the left, given the mass values). Note that the "external force" that motivates "m_1" will be "m_1g" and the "external force" that motivates "m_3" will be "m_3g," and noting by inspection that the normal force on "m_2" is "m_2g," we can deal only with the forces that will actually accelerate the system and write:

$$\sum F_{acc}:$$

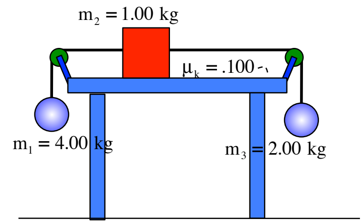
$$m_1g - m_3g - \mu_k N_2 = (m_1 + m_2 + m_3)a$$

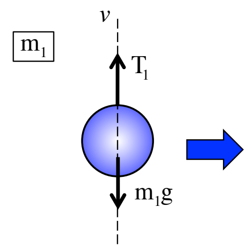
$$\Rightarrow \quad m_1g - m_3g - \mu_k (m_2g) = (m_1 + m_2 + m_3)a$$

$$\Rightarrow \quad a = \frac{m_1g - m_3g - \mu_k (m_2g)}{m_1 + m_2 + m_3} \qquad \text{Equation 1}$$

That's Quick and Dirty! Now let's see if the Formal approach gives us the same acceleration term.

Determine the acceleration of the system using the Formal approach:

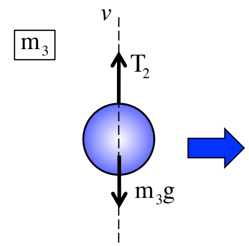




$$\sum F_v:$$

$$T_1 - m_1g = -m_1a$$

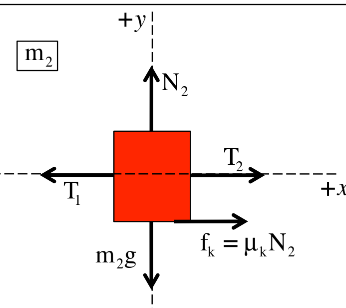
$$\Rightarrow \quad T_1 = m_1g - m_1a \qquad \text{Equation A}$$

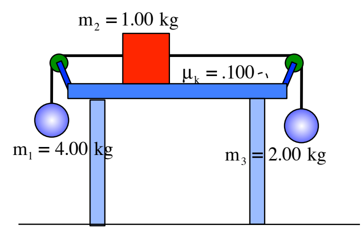


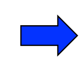
$$\sum F_v:$$

$$T_2 - m_3g = m_3a$$

$$\Rightarrow \quad T_2 = m_3g + m_3a \qquad \text{Equation B}$$







$$\sum F_y:$$

$$N_2 - m_2g = -\cancelto{0}{m_2a}$$

$$\Rightarrow \quad N_2 = m_2g \qquad \text{Equation C}$$

$$\sum F_v:$$

$$-T_1 + T_2 + \mu_k N_2 = -m_2a \qquad \text{Equation D}$$

Note that I've identified signs relative to the coordinate axes system I'm using!

Substituting Equation A, B and C into Equation D, we get:

$$- \quad T_1 \quad + \quad T_2 \quad + \mu_k \quad N_2 \; = -m_2 a$$

$$-(m_1 g - m_1 a) + (m_3 g + m_3 a) + \mu_k (m_2 g) = -m_2 a$$

$$\Rightarrow \quad a = \frac{m_1 g - m_3 g - \mu_k (m_2 g)}{m_1 + m_2 + m_3}$$

Equation E

$$= \frac{(4.00 \text{ kg})(9.80 \text{ m/s}^2) - (2.00 \text{ kg})(9.80 \text{ m/s}^2) - (.350)\big((1.00 \text{ kg})(9.80 \text{ m/s}^2)\big)}{(4.00 + 2.00 + 1.00)\text{kg}}$$

$$= 2.31 \text{ m/s}^2$$

Notice that Equation 1 from the Quick and Dirty approach and Equation E from the Formal approach are the same. Yeeha!

In any case, m_1's acceleration is downward; m_2's acceleration is to the left and m_3's acceleration is upward.

c.) From Equations A and B, the tensions are found to be:

$$T_1 = m_1 g - m_1 a$$

Equation A

$$= (4.00 \text{ kg})(9.80 \text{ m/s}^2) - (4.00 \text{ kg})(2.31 \text{ m/s}^2)$$

$$= 30.0 \text{ N}$$

and

$$T_2 = m_3 g + m_3 a$$

Equation B

$$= (2.00 \text{ kg})(9.80 \text{ m/s}^2) + (2.00 \text{ kg})(2.31 \text{ m/s}^2)$$

$$= 24.2 \text{ N}$$

d.) If friction diminished, would the tensions increase, decrease or stay the same?

Without friction, you'd expect the acceleration to be bigger. According to Equation A, if "a" increases, "T_1" decreases. And according to Equation B, if "a" increases, "T_2" increases. I don't know that this is altogether obvious upon simple inspection. Fortunately, we have the math to help us out.